# FACULTY OF HEALTH SCIENCES

# SPEECH AND LANGUAGE THERAPY

# PHYSICS OF SOUND

## AUTUMN TERM 2023-2024

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# Week 2

# VIBRATION & WAVES

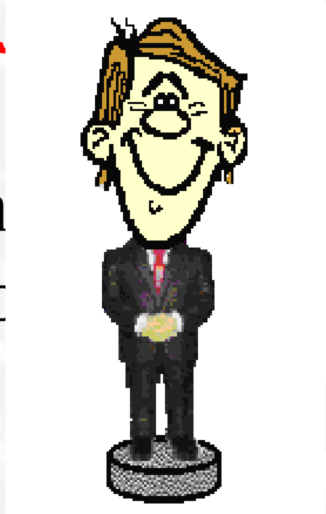
# VIBRATIONS

- Vibrational Motion
- Properties of Periodic Motion
- Pendulum Motion
- Motion of a Mass on a Spring

# Vibrational Motion

Things wiggle. They do *the back and forth*. They vibrate; they shake; they oscillate. These phrases describe the motion of a variety of objects. They even describe the motion of matter at the atomic level. Even atoms wiggle - they do the back and forth. Wiggles, vibrations, and oscillations are an inseparable part of nature. In this chapter of The Physics Classroom Tutorial, we will make an effort to understand vibrational motion and its relationship to waves. An understanding of vibrations and waves is essential to understanding our physical world. Much of what we see and hear is only possible because of vibrations and waves. We see the world around us because of light waves. And we hear the world around us because of sound waves. If we can understand waves, then we will be able to understand the world of sight and sound.

# Bobblehead Dolls - An Example of a Vibrating Obj



To begin our ponderings of vibrations and waves, consider one of th crazy bobblehead dolls that you've likely seen at baseball stadiums c novelty shops. A bobblehead doll consists of an oversized replica of person's head attached by a spring to a body and a stand. A light tap oversized head causes it to bobble. The head wiggles; it vibrates; it oscillates. When pushed or somehow disturbed, the head does *the back and forth*. The back and forth doesn't happen forever. Over time, the vibrations tend to *die off* and the bobblehead stops bobbing and finally assumes its usual resting position.

The bobblehead doll is a good illustration of many of the principles of vibrational motion. Think about how you would describe the back and forth motion of the oversized head of a bobblehead doll. What words would you use to describe such a motion? How does the motion of the bobblehead change over time? How does the motion of one bobblehead differ from the motion of another bobblehead? What quantities could you measure to describe the motion and so distinguish one motion from another motion? How would you explain the cause of such a motion? Why does the back and forth motion of the bobblehead finally stop? These are all questions worth pondering and answering if we are to understand vibrational motion. These are the questions we will attempt to answer in Section 1 of this chapter.

# What Causes Objects to Vibrate?

Like any object that undergoes vibrational motion, the bobblehead has a **resting position**. The resting position is the position assumed by the bobblehead when it is not vibrating. The resting position is sometimes referred to as the **equilibrium position**. When an object is positioned at its equilibrium position, it is in **a state of equilibrium**.

As discussed in the Newton's Law Chapter of the Tutorial, an object which is in a state of equilibrium is experiencing a balance of forces. All the individual forces - gravity, spring, etc. - are balanced or add up to an overall net force of 0 Newtons. When a bobblehead is at the equilibrium position, the forces on the bobblehead are balanced. The bobblehead will remain in this position until somehow disturbed from its equilibrium.

When in its resting position, the bobblehead is at equilibrium; all the forces acting upon it are balanced. When a force is applied to the bobblehead, it is displaced from its equilibrium position. This force disturbs the equilibrium and is the cause of the bobblehead's vibration.

If a force is applied to the bobblehead, the equilibrium will be disturbed and the bobblehead will begin vibrating. We could use the phrase forced vibration to describe the force which sets the otherwise resting bobblehead into motion. In this case, the force is a short-lived, momentary force that begins the motion. The bobblehead does its back and forth, repeating the motion over and over. Each repetition of its back and forth motion is a little less vigorous than its previous repetition.

If the head sways 3 cm to the right of its equilibrium position during the first repetition, it may only sway 2.5 cm to the right of its equilibrium position during the second repetition. And it may only sway 2.0 cm to the right of its equilibrium position during the third repetition. And so on. The extent of its displacement from the equilibrium position becomes less and less over time. Because the forced vibration that initiated the motion is a single instance of a short-lived, momentary force, the vibrations ultimately cease. The bobblehead is said to experience damping.

Damping is the tendency of a vibrating object to lose or to dissipate its energy over time. The mechanical energy of the bobbing head is lost to other objects. Without a sustained forced vibration, the back and forth motion of the bobblehead eventually ceases as energy is dissipated to other objects. A sustained input of energy would be required to keep the back and forth motion going. After all, if the vibrating object naturally loses energy, then it must continuously be put back into the system through a forced vibration in order to sustain the vibration.

# The Restoring Force

A vibrating bobblehead often does the back and forth a number of times. The vibrations repeat themselves over and over. As such, the bobblehead will move back to (and past) the equilibrium position every time it returns from its maximum displacement to the right or the left (or above or below). This begs a question - and perhaps one that you have been thinking of yourself as you've pondered the topic of vibration.

If the forces acting upon the bobblehead are balanced when at the equilibrium position, then why does the bobblehead sway past this position?

Why doesn't the bobblehead stop the first time it returns to the equilibrium position?

The answer to this question can be found in Newton's first law of motion. Like any moving object, the motion of a vibrating object can be understood in light of Newton's laws. According to Newton's law of inertia, an object which is moving will continue its motion if the forces are balanced. Put another way, forces, when balanced, do not stop moving objects.

So every instant in time that the bobblehead is at the equilibrium position, the momentary balance of forces will not stop the motion. The bobblehead keeps moving. It moves past the equilibrium position towards the opposite side of its swing. As the bobblehead is displaced past its equilibrium position, then a force capable of slowing it down and stopping it exists. This force that slows the bobblehead down as it moves away from its equilibrium position is known as a restoring force. The restoring force acts upon the vibrating object to move it back to its original equilibrium position.

Vibrational motion is often contrasted with translational motion. In translational motion, an object is permanently displaced. The initial force that is imparted to the object displaces it from its resting position and sets it into motion. Yet because there is no restoring force, the object continues the motion in its original direction. When an object vibrates, it doesn't move permanently out of position. The restoring force acts to slow it down, change its direction and force it back to its original equilibrium position. An object in translational motion is permanently displaced from its original position. But an object in vibrational motion wiggles about a fixed position - its original equilibrium position. Because of the restoring force, vibrating objects do the back and forth.

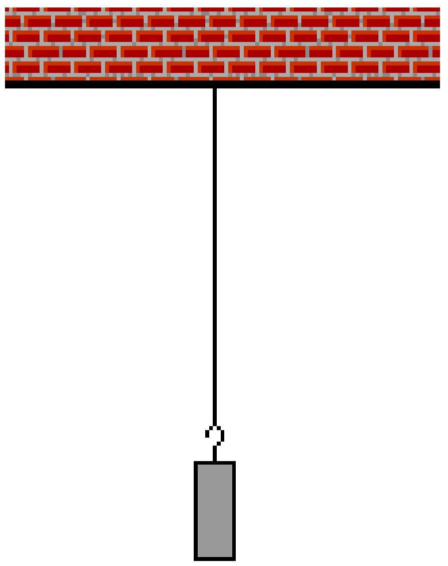
BİRUNİ ÜNİVERSİTESİ "Bilimin Geleceği"

# Other Vibrating Systems

As you know, bobblehead dolls are not the only objects that vibrate. It might be safe to say that all objects in one way or another can be forced to vibrate to some extent. The vibrations might not be large enough to be visible. Or the amount of damping might be so strong that the object scarcely completes a full cycle of vibration. But as long as a force persists to restore the object to its original position, a displacement from its resting position will result in a vibration.

Even a large massive skyscraper is known to vibrate as winds push upon its structure. While held fixed in place at its foundation (we hope), the winds force the length of the structure out of position and the skyscraper is forced into vibration.

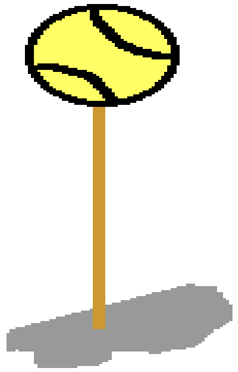
A pendulum is a classic example of an object that is considered to vibrate. A simple pendulum consists of a relatively massive object hung by a string from a fixed support. It typically hangs vertically in its equilibrium position. When the mass is displaced from equilibrium, it begins its back and forth vibration about its fixed equilibrium position. The motion is regular and repeating. Because of the regular nature of a pendulum's motion, many clocks, such as grandfather clocks, use a pendulum as part of its timing mechanism.



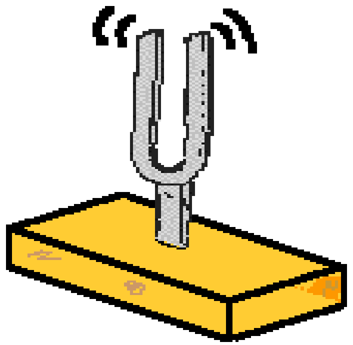
The vibrational motion of a pendulum is regular and repeating.

An inverted pendulum is another classic example of an object that undergoes vibrational motion. An inverted pendulum is simply a pendulum which has its fixed end located below the vibrating mass. An inverted pendulum can be made by attaching a mass (such as a tennis ball) to the top end of a dowel rod and then securing the bottom end of the dowel rod to a horizontal support.

This is shown in the diagram below. A gentle force exerted upon the tennis ball will cause it to vibrate about a fixed, equilibrium position. The vibrating skyscraper can be thought of as a type of inverted pendulum. Tall trees are often displaced from their usual vertical orientation by strong winds. As the winds cease, the trees will vibrate back and forth about their fixed positions. Such trees can be thought of as acting as inverted pendula. Even the tines of a tuning fork can be considered a type of inverted pendulum

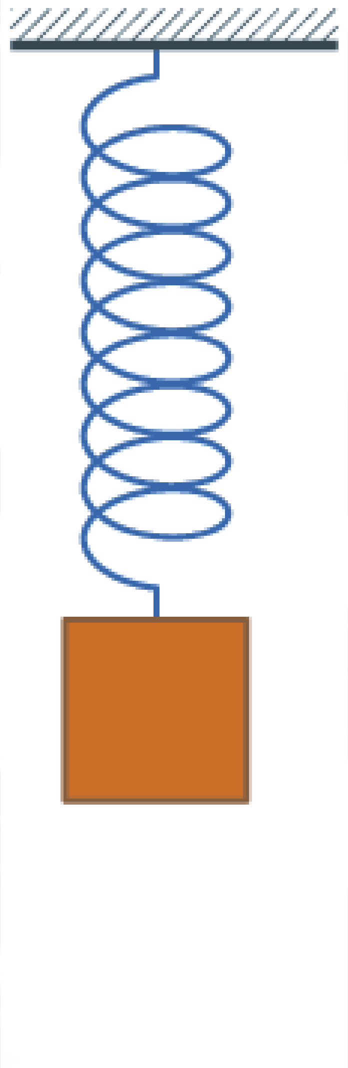


A mass attached to the top of a flexible rod forms an inverted pendulum.



The tines of a tuning fork vibrate; its base is fixed. This is another example of an inverted pendulum.

Another classic example of an object that undergoes vibrational motion is a mass on a spring. The animation at the right depicts a mass suspended from a spring. The mass hangs at a resting position. If the mass is pulled down, the spring is stretched. Once the mass is released, it begins to vibrate.



It does the back and forth, vibrating about a fixed position. If the spring is rotated horizontally and the mass is placed upon a supporting surface, the same back and forth motion can be observed. Pulling the mass to the right of its resting position stretches the spring. When released, the mass is pulled back to the left, heading towards its resting position. After passing by its resting position, the spring begins to compress. The compressions of the coiled spring result in a restoring force that again pushes rightward on the leftward moving mass. The cycle continues as the mass vibrates back and forth about a fixed position. The springs inside of a bed mattress, the suspension systems of some cars, and bathroom scales all operated as a mass on a spring system.

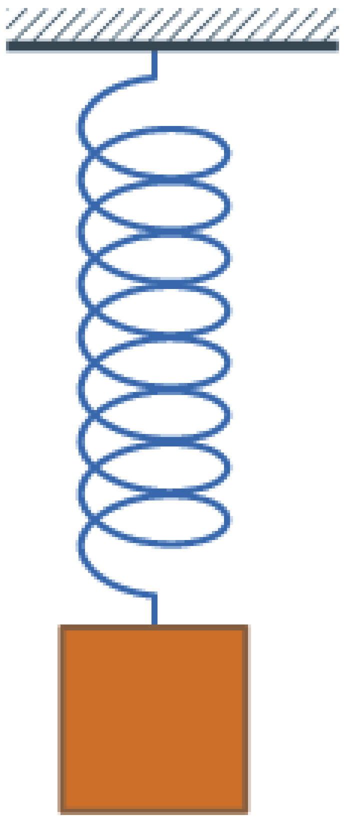
In all the vibrating systems just mentioned, damping is clearly evident. The simple pendulum doesn't vibrate forever; its energy is gradually dissipated through air resistance and loss of energy to the support. The inverted pendulum consisting of a tennis ball mounted to the top of a dowel rod does not vibrate forever. Like the simple pendulum, the energy of the tennis ball is dissipated through air resistance and vibrations of the support. Frictional forces also cause the mass on a spring to lose its energy to the surroundings. In some instances, damping is a favored feature. Car suspension systems are intended to dissipate vibrational energy, preventing drivers and passengers from having to do the back and forth as they also do the down the road.

Hopefully a lot of our original questions have been answered. But one question that has not yet been answered is the question pertaining to quantities that can be measured. How can we quantitatively describe a vibrating object? What measurements can be made of vibrating objects that would distinguish one vibrating object from another? We will ponder this question in the next part of this lesson on vibrational motion.

# Properties of Periodic Motion

# Properties of Periodic Motion

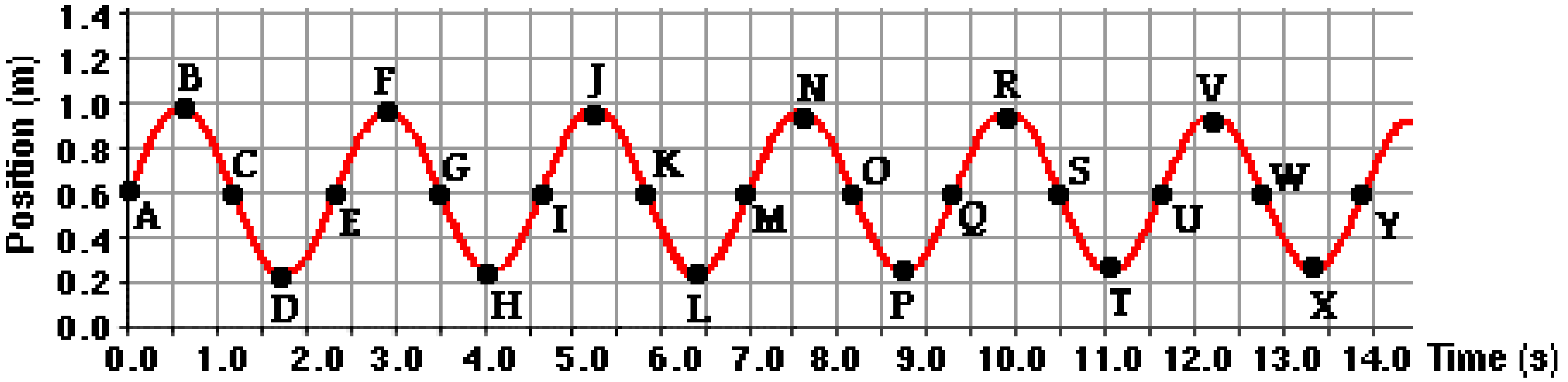
A vibrating object is wiggling about a fixed position. Like the mass on a spring in the animation at the right, a vibrating object is moving over the same path over the course of time. Its motion repeats itself over and over again. If it were not for damping, the vibrations would endure forever (or at least until someone catches the mass and brings it to rest). The mass on the spring not only repeats the same motion, it does so in a regular fashion. The time it takes to complete one back and forth cycle is always the same amount of time.



If it takes the mass 3.2 seconds for the mass to complete the first back and forth cycle, then it will take 3.2 seconds to complete the seventh back and forth cycle. It's like clockwork. It's so predictable that you could set your watch by it. In Physics, a motion that is regular and repeating is referred to as a periodic motion. Most objects that vibrate do so in a regular and repeated fashion; their vibrations are periodic. (Special thanks to Oleg Alexandrov for the animation of the mass on a spring. It is a public domain acquired from WikiMedia Commons. )

# The Sinusoidal Nature of a Vibration

Suppose that a motion detector was placed below a vibrating mass on a spring in order to detect the changes in the mass's position over the course of time. And suppose that the data from the motion detector could represent the motion of the mass by a position vs. time plot. The graphic below depicts such a graph. For discussion sake, several points have been labeled on the graph to assist in the follow-up discussion.



Before reading on, take a moment to reflect on the type of information that is conveyed by the graph. And take a moment to reflect about what quantities on the graph might be important in understanding the mathematical description of a mass on a spring. If you've taken time to ponder these questions, the following discussion will likely be more meaningful.

One obvious characteristic of the graph has to do with its shape. Many students recognize the shape of this graph from experiences in Mathematics class. The graph has the shape of a sine wave. If y = sine(x) is plotted on a graphing calculator, a graph with this same shape would be created. The vertical axis of the above graph represents the position of the mass relative to the motion detector. A position of about 0.60 m cm above the detector represents the resting position of the mass. So the mass is vibrating back and forth about this fixed resting position over the course of time. There is something sinusoidal about the vibration of a mass on a spring. And the same can be said of a pendulum vibrating about a fixed position or of a guitar string or of the air inside of a wind instrument. The position of the mass is a function of the sine of the time.

A second obvious characteristic of the graph may be its periodic nature. The motion repeats itself in a regular fashion. Time is being plotted along the horizontal axis; so any measurement taken along this axis is a measurement of the time for something to happen. A full cycle of vibration might be thought of as the movement of the mass from its resting position (A) to its maximum height (B), back down past its resting position (C) to its minimum position (D), and then back to its resting position (E). Using measurements from along the time axis, it is possible to determine the time for one complete cycle. The mass is at position A at a time of 0.0 seconds and completes its cycle when it is at position E at a time of 2.3 seconds. It takes 2.3 seconds to complete the first full cycle of vibration.

Now if the motion of this mass is periodic (i.e., regular and repeating), then it should take the same time of 2.3 seconds to complete any full cycle of vibration. The same time-axis measurements can be taken for the sixth full cycle of vibration. In the sixth full cycle, the mass moves from a resting position (U) up to V, back down past W to X and finally back up to its resting position (Y) in the time interval from 11.6 seconds to 13.9 seconds.

This represents a time of 2.3 seconds to complete the sixth full cycle of vibration. The two cycle times are identical. Other cycle times are indicated in the table below. By inspection of the table, one can safely conclude that the motion of the mass on a spring is regular and repeating; it is clearly periodic. The small deviation from 2.3 s in the third cyle can be accounted for by the lack of precision in the reading of the graph.

| Cycle | Letters | Times at Beginning and End of Cycle (seconds) | Cycle Time (seconds) |
|---|---|---|---|
| 1st | A to E | 0.0 sto 2.3 s | 2.3 |
| 2nd | E tp I | 2.3 s to 4.6 s | 2.3 |
| 3rd | I to M | 4.6 s to 7.0 s | 2.4 |
| 4th | M to Q | 7.0 s to 9.3 s | 2.3 |
| 5th | Q to U | 9.3 s to 11.6 s | 2.3 |
| 6th | U to Y | 11.6 s to 13.9 s | 2.3 |

A third obvious characteristic of the graph is that damping occurs with the mass-spring system. Some energy is being dissipated over the course of time. The extent to which the mass moves above (B, F, J, N, R and V) or below (D, H, L, P, T and X) the resting position (C, E, G, I, etc.) varies over the course of time. In the first full cycle of vibration being shown, the mass moves from its resting position (A) 0.60 m above the motion detector to a high position (B) of 0.99 m cm above the motion detector. This is a total upward displacement of 0.29 m. In the sixth full cycle of vibration that is shown, the mass moves from its resting position (U) 0.60 m above the motion detector to a high position (V) 0.94 m above the motion detector. This is a total upward displacement of 0.24 m cm. The table below summarizes displacement measurements for several other cycles displayed on the graph.

| Cycle | Letters | Maximum Upward Displacement | Maximum Downward Displacement |
|---|---|---|---|
| 1st | A to E | 0.60 m to 0.99 m | 0.60 m to 0.21 m |
| 2nd | E to I | 0.60 m to 0.98 m | 0.60 m to 0.22 m |
| 3rd | I to M | 0.60 m to 0.97 m | 0.60 m to 0.23 m |
| 6th | U to Y | 0.60 m to 0.94 m | 0.60 m to 0.26 m |

Over the course of time, the mass continues to vibrate - moving away from and back towards the original resting position. However, the amount of displacement of the mass at its maximum and minimum height is decreasing from one cycle to the next. This illustrates that energy is being lost from the mass-spring system. If given enough time, the vibration of the mass will eventually cease as its energy is dissipated.

Perhaps, this observation of energy dissipation or energy loss is the observation that triggers the "slowing down" comment discussed earlier. In physics (or at least in the English language), "slowing down" means to "get slower" or to "lose speed". Speed, a physics term, refers to how fast or how slow an object is moving. To say that the mass on the spring is "slowing down" over time is to say that its speed is decreasing over time. But as mentioned (and as will be discussed in great detail later), the mass speeds up during two intervals of every cycle. As the restoring force pulls the mass back towards its resting position (for instance, from B to C and from D to E), the mass speeds up.

For this reason, a physicist adopts a different language to communicate the idea that the vibrations are "dying out". We use the phrase "energy is being dissipated or lost" instead of saying the "mass is slowing down." Language is important when it comes to learning physics. And sometimes, faulty language (combined with surface-level thinking) can confuse a student of physics who is sincerely trying to learn new ideas.

# Period and Frequency

So far in this part of the lesson, we have looked at measurements of time and position of a mass on a spring. The measurements were based upon readings of a position-time graph. The data on the graph was collected by a motion detector that was capturing a history of the motion over the course of time. The key measurements that have been made are measurements of:the time for the mass to complete a cycle, andthe maximum displacement of the mass above (or below) the resting position.These two measurable quantities have names. We call these quantities period and amplitude.

An object that is in periodic motion - such as a mass on a spring, a pendulum or a bobblehead doll - will undergo back and forth vibrations about a fixed position in a regular and repeating fashion. The fact that the periodic motion is regular and repeating means that it can be mathematically described by a quantity known as the period. The period of the object's motion is defined as the time for the object to complete on Being a time, the period is measured in units such as seconds ds, days or even years. The standard metric unit for period is

An object in periodic motion can have a long period or a short period. For instance, a pendulum bob tied to a 1-meter length string has a period of about 2.0 seconds. For comparison sake, consider the vibrations of a piano string that plays the middle C note (the C note of the fourth octave). Its period is approximately 0.0038 seconds (3.8 milliseconds). When comparing these two vibrating objects - the 1.0-meter length pendulum and the piano string which plays the middle C note - we would describe the piano string as vibrating relatively frequently and we would describe the pendulum as vibrating relatively infrequently. Observe that the description of the two objects uses the terms frequently and infrequently. The terms fast and slow are not used since physics types reserve the words fast and slow to refer to an object's speed.

Here in this description we are referring to the frequency, not the speed. An object can be in periodic motion and have a low frequency and a high speed. As an example, consider the periodic motion of the moon in orbit about the earth. The moon moves very fast; its orbit is highly infrequent. It moves through space with a speed of about 1000 m/s - that's fast. Yet it makes a complete cycle about the earth once every 27.3 days (a period of about 2.4x105 seconds) - that's infrequent.

Objects like the piano string that have a relatively short period (i.e., a low value for period) are said to have a high frequency. Frequency is another quantity that can be used to quantitatively describe the motion of an object is periodic motion. The frequency is defined as the number of complete cycles occurring per period of time. Since the standard metric unit of time is the second, frequency has units of cycles/second. The unit cycles/second is equivalent to the unit Hertz (abbreviated Hz). The unit Hertz is used in honor of Heinrich Rudolf Hertz, a 19th century physicist who expanded our understanding of the electromagnetic theory of light waves.

The concept and quantity frequency is best understood if you attach it to the everyday English meaning of the word. Frequency is a word we often use to describe how often something occurs. You might say that you frequently check your email or you frequently talk to a friend or you frequently wash your hands when working with chemicals. Used in this context, you mean that you do these activities often. To say that you frequently check your email means that you do it several times a day - you do it often. In physics, frequency is used with the same meaning - it indicates how often a repeated event occurs. High frequency events that are periodic occur often, with little time in between each occurrence - like the back and forth vibrations of the tines of a tuning fork. The vibrations are so frequent that they can't be seen with the naked eye. A 256-Hz tuning fork has tines that make 256 complete back and forth vibrations each second. At this frequency, it only takes the tines about 0.00391 seconds to complete one cycle. A 512-Hz tuning fork has an even higher frequency. Its vibrations occur more frequently; the time for a full cycle to be completed is 0.00195 seconds. In comparing these two tuning forks, it is obvious that the tuning fork with the highest frequency has the lowest period.

The two quantities frequency and period are inversely related to each other. In fact, they are mathematical reciprocals of each other. The frequency is the reciprocal of the period and the period is the reciprocal of the frequency.

$$\text{period} = \frac{1}{\text{frequency}} \qquad \text{frequency} = \frac{1}{\text{period}}$$

Consider their definitions as restated below:

period = the time for one full cycle to complete itself; i.e., seconds/cycle

frequency = the number of cycles that are completed per time; i.e., cycles/second

Even the definitions have a reciprocal ring to them. To understand the distinction between period and frequency, consider the following statement:According to Wikipedia (and as of this writing), Tim Ahlstrom of Oconomowoc, WI holds the record for hand clapping. He is reported to have clapped his hands 793 times in 60.0 seconds.What is the frequency and what is the period of Mr. Ahlstrom's hand clapping during this 60.0-second period?

In this problem, the event that is repeating itself is the clapping of hands; one hand clap is equivalent to *a cycle*.

Frequency = cycles per second = 793 cycles/60.0 seconds = 13.2 cycles/s
= **13.2 Hz**

Period = seconds per cycle = 60.0 s/793 cycles = **0.0757 seconds**

# Amplitude of Vibration

The final measurable quantity that describes a vibrating object is the amplitude. The amplitude is defined as the maximum displacement of an object from its *resting position*. The resting position is that position assumed by the object when not vibrating. Once vibrating, the object oscillates about this fixed position. If the object is a mass on a spring (such as the discussion earlier on this page), then it might be displaced a maximum distance of 35 cm below the resting position and 35 cm above the resting position. In this case, the amplitude of motion is 35 cm.

Over the course of time, the amplitude of a vibrating object tends to become less and less. The amplitude of motion is a reflection of the quantity of energy possessed by the vibrating object. An object vibrating with a relatively large amplitude has a relatively large amount of energy. Over time, some of this energy is lost due to damping. As the energy is lost, the amplitude decreases. If given enough time, the amplitude decreases to 0 as the object finally stops vibrating. At this point in time, it has lost all its energy.

# Pendulum Motion

A simple pendulum consists of a relatively massive object hung by a string from a fixed support. It typically hangs vertically in its equilibrium position. The massive object is affectionately referred to as the *pendulum bob*. When the bob is displaced from equilibrium and then released, it begins its back and forth vibration about its fixed equilibrium position. The motion is regular and repeating, an example of periodic motion. Pendulum motion was introduced earlier in this lesson as we made an attempt to understand the nature of vibrating objects.

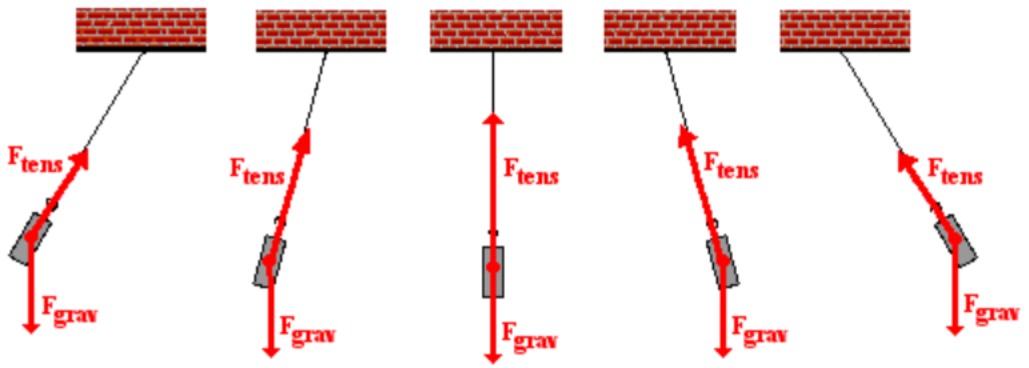
Pendulum motion was discussed again as we looked at the mathematical properties of objects that are in periodic motion. Here we will investigate pendulum motion in even greater detail as we focus upon how a variety of quantities change over the course of time. Such quantities will include forces, position, velocity and energy - both kinetic and potential energy.

# Force Analysis of a Pendulum

Earlier in this lesson we learned that an object that is vibrating is acted upon by a restoring force. The restoring force causes the vibrating object to slow down as it moves away from the equilibrium position and to speed up as it approaches the equilibrium position. It is this restoring force that is responsible for the vibration. So what forces act upon a pendulum bob? And what is the restoring force for a pendulum? There are two dominant forces acting upon a pendulum *bob* at all times during the course of its motion. There is the force of gravity that acts downward upon the bob. It results from the Earth's mass attracting the mass of the bob. And there is a tension force acting upward and towards the pivot point of the pendulum. The tension force results from the string pulling upon the *bob* of the pendulum.

In our discussion, we will *ignore* the influence of air resistance - a third force that always opposes the motion of the bob as it swings to and fro. The air resistance force is relatively weak compared to the two dominant forces.

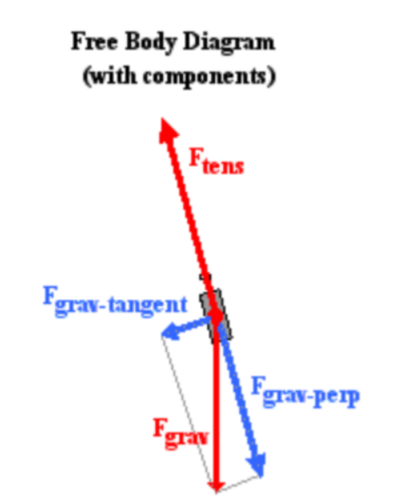
The gravity force is highly predictable; it is always in the same direction (down) and always of the same magnitude - mass*9.8 N/kg. The tension force is considerably less predictable. Both its direction and its magnitude change as the bob swings to and fro. The direction of the tension force is always towards the pivot point. So as the bob swings to the left of its equilibrium position, the tension force is at an angle - directed upwards and to the right. And as the bob swings to the right of its equilibrium position, the tension is directed upwards and to the left. The diagram below depicts the direction of these two forces at five different positions over the course of the pendulum's path.




The force of gravity is always directed downward; its magnitude never changes. The tension force is always directed towards the pivot; it's magnitude varies over the course of a vibrational cycle.

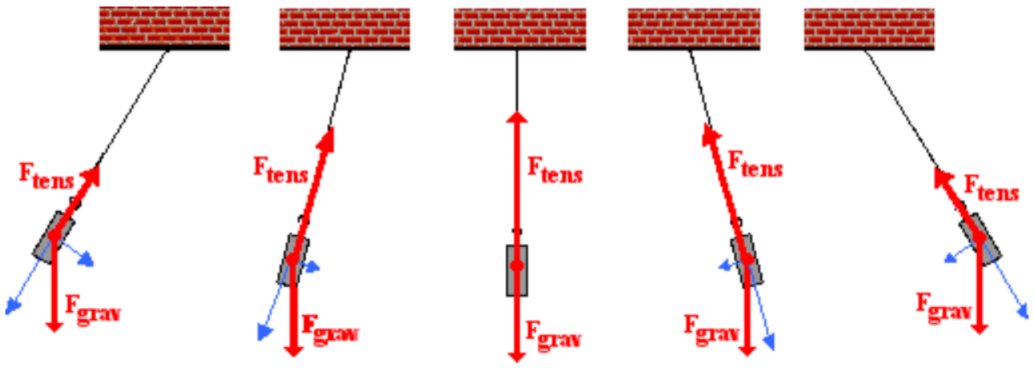
In physical situations in which the forces acting on an object are not in the same, opposite or perpendicular directions, it is customary to resolve one or more of the forces into components. This was the practice used in the analysis of sign hanging problems and inclined plane problems. Typically one or more of the forces are resolved into perpendicular components that lie along coordinate axes that are directed in the direction of the acceleration or perpendicular to it. So in the case of a pendulum, it is the gravity force which gets resolved since the tension force is already directed perpendicular to the motion. The diagram at the right shows the pendulum bob at a position to the right of its equilibrium position and midway to the point of maximum displacement.

A coordinate axis system is sketched on the diagram and the force of gravity is resolved into two components that lie along these axes. One of the components is directed tangent to the circular arc along which the pendulum bob moves; this component is labeled Fgrav-tangent. The other component is directed perpendicular to the arc; it is labe rp. You will notice that the perpendicular component of grav pposite direction of the tension force. You might also notice that rce is slightly larger than this component of gravity. The fact th force (Ftens) is greater than the perpendicular component of grav rp) means there will be a net force which is perpendicular to the arc of the bob's motion.



This must be the case since we expect that objects that move along circular paths will experience an inward or centripetal force. The tangential component of gravity (Fgrav-tangent) is unbalanced by any other force. So there is a net force directed along the other coordinate axes. It is this tangential component of gravity which acts as the restoring force. As the pendulum bob moves to the right of the equilibrium position, this force component is directed opposite its motion back towards the equilibrium position.

The above analysis applies for a single location along the pendulum's arc. At the other locations along the arc, the strength of the tension force will vary. Yet the process of resolving gravity into two components along axes that are perpendicular and tangent to the arc remains the same. The diagram below shows the results of the force analysis for several other positions.

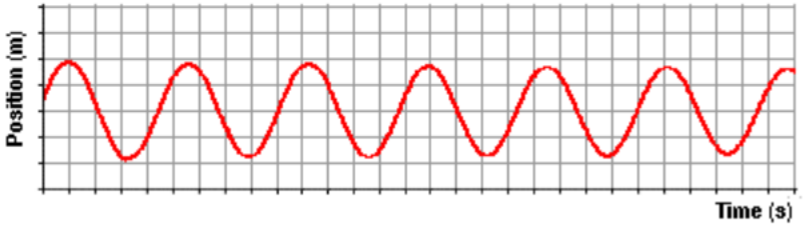


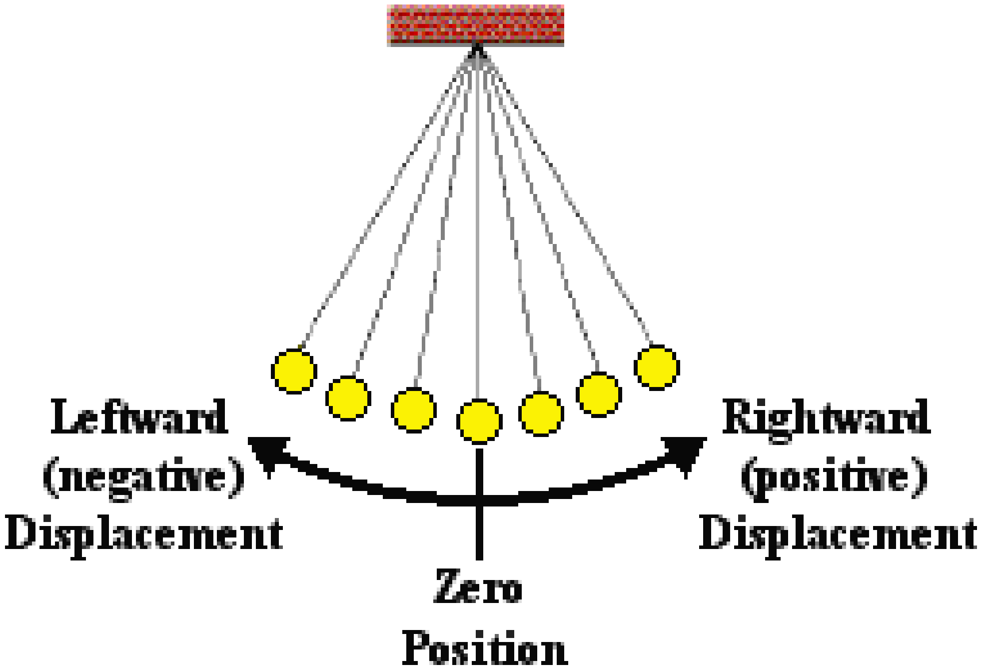
There are a couple comments to be made. First, observe the diagram for when the bob is displaced to its maximum displacement to the right of the equilibrium position. This is the position in which the pendulum bob momentarily has a velocity of 0 m/s and is changing its direction. The tension force (Ftens) and the perpendicular component of gravity (Fgrav-perp) balance each other. At this instant in time, there is no net force directed along the axis that is perpendicular to the motion. Since the motion of the object is momentarily paused, there is no need for a centripetal force.

Second, observe the diagram for when the bob is at the equilibrium position (the string is completely vertical). When at this position, there is no component of force along the tangent direction. When moving through the equilibrium position, the restoring force is momentarily absent. Having been restored to the equilibrium position, there is no restoring force. The restoring force is only needed when the pendulum bob has been displaced away from the equilibrium position. You might also notice that the tension force (Ftens) is greater than the perpendicular component of gravity (Fgrav-perp) when the bob moves through this equilibrium position. Since the bob is in motion along a circular arc, there must be a net centripetal force at this position.

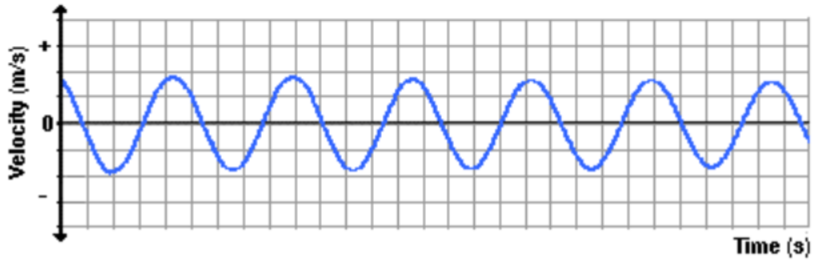
# The Sinusoidal Nature of Pendulum Motion

In the previous part of this lesson, we investigated the sinusoidal nature of the motion of a mass on a spring. We will conduct a similar investigation here for the motion of a pendulum bob. Let's suppose that we could measure the amount that the pendulum bob is displaced to the left or to the right of its equilibrium or rest position over the course of time. A displacement to the right of the equilibrium position would be regarded as a positive displacement; and a displacement to the left would be regarded as a negative displacement. Using this reference frame, the equilibrium position would be regarded as the zero position. And suppose that we constructed a plot showing the variation in position with respect to time. The resulting position vs. time plot is shown below. Similar to what was observed for the mass on a spring, the position of the pendulum bob (measured along the arc relative to its rest position) is a function of the sine of the time.

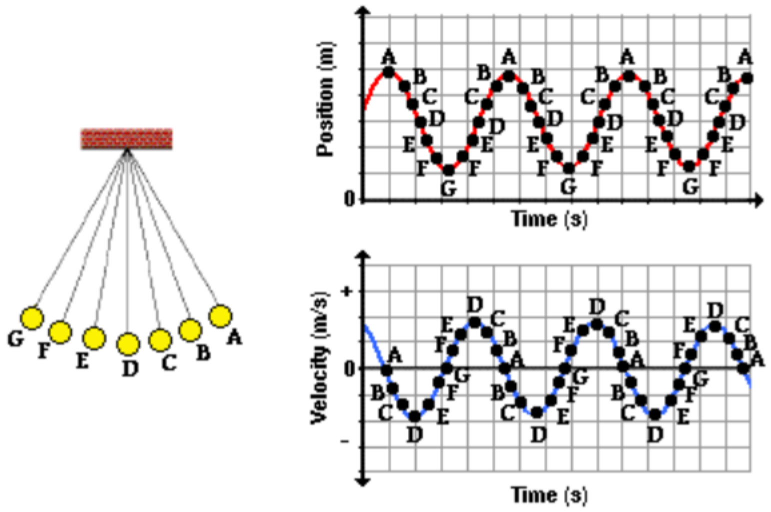

Position (m)
Time (s)


Leftward (negative) Displacement
Rightward (positive) Displacement
Zero Position

Now suppose that we use our motion detector to investigate the how the velocity of the pendulum changes with respect to the time. As the pendulum bob does the back and forth, the velocity is continuously changing. There will be times at which the velocity is a negative value (for moving leftward) and other times at which it will be a positive value (for moving rightward). And of course there will be moments in time at which the velocity is 0 m/s. If the variations in velocity over the course of time were plotted, the resulting graph would resemble the one shown below.



Now let's try to understand the relationship between the position of the bob along the arc of its motion and the velocity with which it moves. Suppose we identify several locations along the arc and then relate these positions to the velocity of the pendulum bob. The graphic below shows an effort to make such a connection between position and velocity.

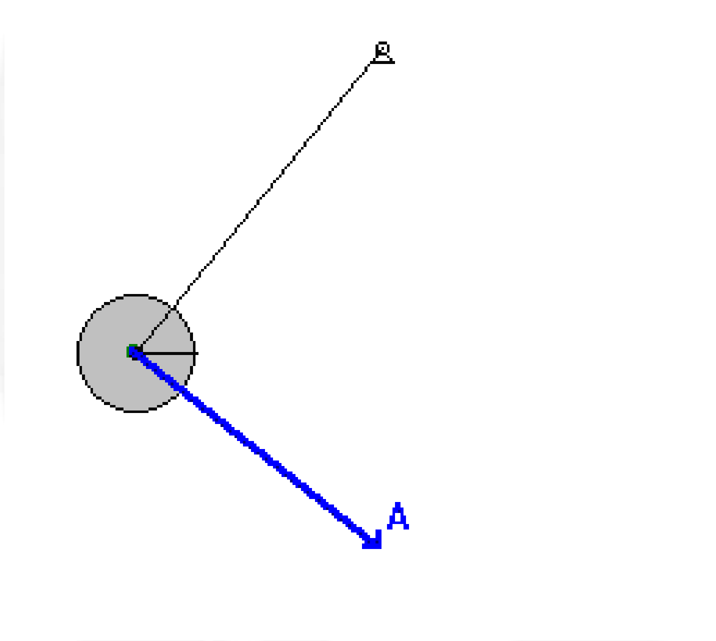


As is often said, a picture is worth a thousand words. Now here come the words. The plot above is based upon the equilibrium position (D) being designated as the zero position. A displacement to the left of the equilibrium position is regarded as a negative position. A displacement to the right is regarded as a positive position. An analysis of the plots shows that the velocity is least when the displacement is greatest. And the velocity is greatest when the displacement of the bob is least. The further the bob has moved away from the equilibrium position, the slower it moves; and the closer the bob is to the equilibrium position, the faster it moves. This can be explained by the fact that as the bob moves away from the equilibrium position, there is a restoring force that opposes its motion. This force slows the bob down. So as the bob moves leftward from position D to E to F to G, the force and acceleration is directed rightward and the velocity decreases as it moves along the arc from D to G. At G - the maximum displacement to the left - the pendulum bob has a velocity of

You might think of the bob as being momentarily paused and ready to change its direction. Next the bob moves rightward along the arc from G to F to E to D. As it does, the restoring force is directed to the right in the same direction as the bob is moving. This force will accelerate the bob, giving it a maximum speed at position D - the equilibrium position.

As the bob moves past position D, it is moving rightward along the arc towards C, then B and then A. As it does, there is a leftward restoring force opposing its motion and causing it to slow down. So as the displacement increases from D to A, the speed decreases due to the opposing force. Once the bob reaches position A - the maximum displacement to the right - it has attained a velocity of 0 m/s. Once again, the bob's velocity is least when the displacement is greatest. The bob completes its cycle, moving leftward from A to B to C to D. Along this arc from A to D, the restoring force is in the direction of the motion, thus speeding the bob up. So it would be logical to conclude that as the position decreases (along the arc from A to D), the velocity increases. Once at position D, the bob will have a zero displacement and a maximum velocity. The velocity is greatest when the displacement is least.

The animation at the right (used with the permission of Wikimedia Commons; special thanks to Hubert Christiaen) provides a visual depiction of these principles. The acceleration vector that is shown combines both the perpendicular and the tangential accelerations into a single vector. You will notice that this vector is entirely tangent to the arc when at maximum displacement; this is consistent with the force analysis discussed above. And the vector is vertical (towards the center of the arc) when at the equilibrium position. This also is consistent with the force analysis discussed above.



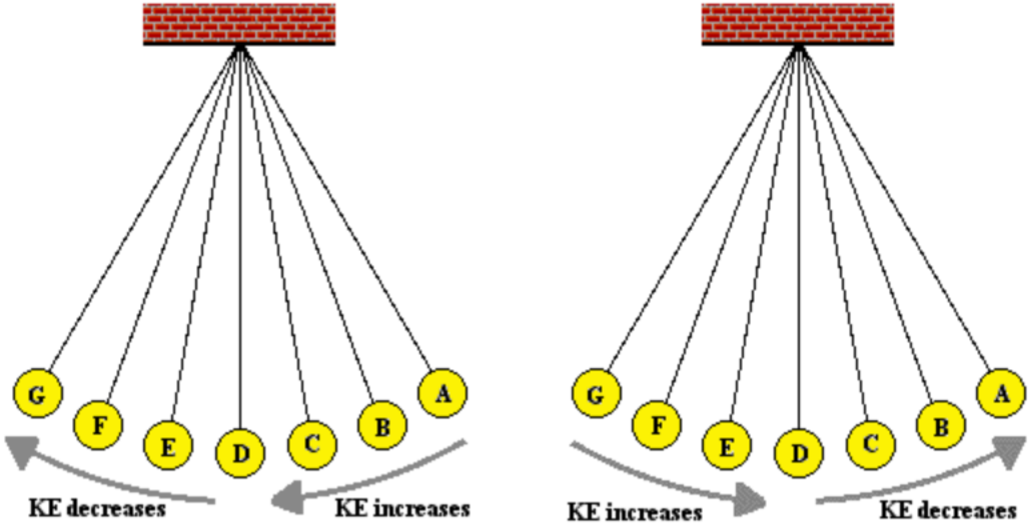
# Energy Analysis

In a previous chapter of The Physics Classroom Tutorial, the energy possessed by a pendulum bob was discussed. We will expand on that discussion here as we make an effort to associate the motion characteristics described above with the concepts of kinetic energy, potential energy and total mechanical energy.

The kinetic energy possessed by an object is the energy it possesses due to its motion. It is a quantity that depends upon both mass and speed. The equation that relates kinetic energy (KE) to mass (m) and speed (v) is

$$KE = ½ \bullet m \bullet v^2$$

The faster an object moves, the more kinetic energy that it will possess. We can combine this concept with the discussion above about how speed changes during the course of motion. This blending of concepts would lead us to conclude that the kinetic energy of the pendulum bob increases as the bob approaches the equilibrium position. And the kinetic energy decreases as the bob moves further away from the equilibrium position.

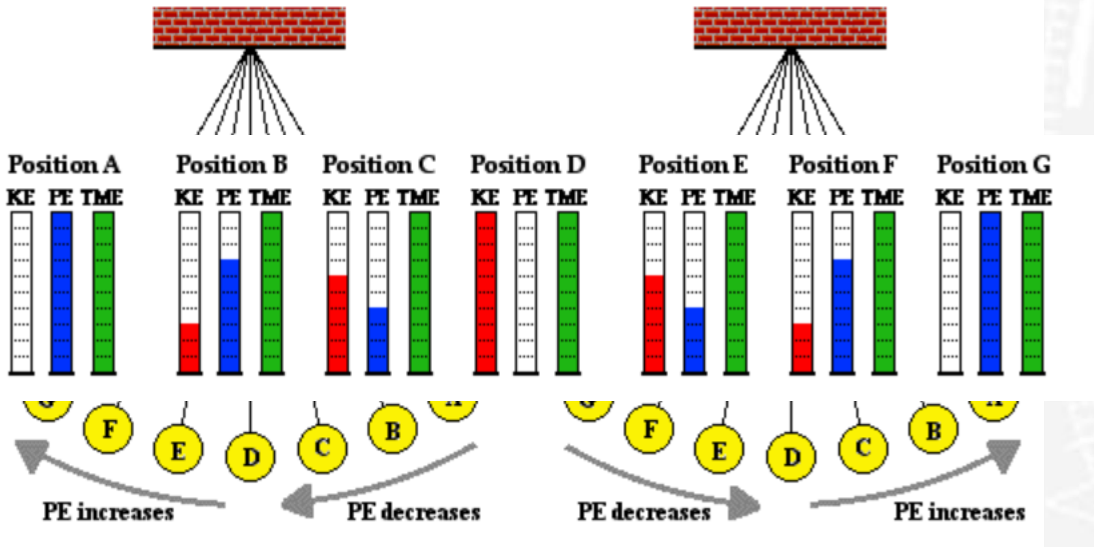

G
F
E
D
C
B
A
KE decreases
KE increases
G
F
E
D
C
B
A
KE increases
KE decreases

The potential energy possessed by an object is the stored energy of position. Two types of potential energy are discussed in The Physics Classroom Tutorial - gravitational potential energy and elastic potential energy. Elastic potential energy is only present when a spring (or other elastic medium) is compressed or stretched. A simple pendulum does not consist of a spring. The form of potential energy possessed by a pendulum bob is gravitational potential energy. The amount of gravitational potential energy is dependent upon the mass (**m**) of the object and the height (**h**) of the object. The equation for gravitational potential energy (**PE**) is

$$PE = m \bullet g \bullet h$$

where **g** represents the gravitational field strength (sometimes referred to as the acceleration caused by gravity) and has the value of 9.8 N/kg.

The height of an object is expressed relative to some arbitrarily assigned zero level. In other words, the height must be measured as a vertical distance above some reference position. For a pendulum bob, it is customary to call the lowest position the reference position or the zero level. So when the bob is at the equilibrium position (the lowest position), its height is zero and its potential energy is 0 J. As the pendulum bob does the back and forth, there are times during which the bob is moving away from the equilibrium position. As it does, its height is increasing as it moves further and further away. It reaches a maximum height as it reaches the position of maximum displacement from the equilibrium position. As the bob moves towards its equilibrium position, it decreases its height and decreases its potential energy.



Position A
KE PE TME
Position B
KE PE TME
Position C
KE PE TME
Position D
KE PE TME
Position E
KE PE TME
Position F
KE PE TME
Position G
KE PE TME
F
E
D
C
B
F
E
D
C
B
PE increases
PE decreases
PE decreases
PE increases

Now let's put these two concepts of kinetic energy and potential energy together as we consider the motion of a pendulum bob moving along the arc shown in the diagram at the right. We will use an energy bar chart to represent the changes in the two forms of energy. The amount of each form of energy is represented by a bar. The height of the bar is proportional to the amount of that form of energy. In addition to the potential energy (**PE**) bar and kinetic energy (**KE**) bar, there is a third bar labeled **TME**. The TME bar represents the total amount of mechanical energy possessed by the pendulum bob. The total mechanical energy is simply the sum of the two forms of energy – kinetic plus potential energy. Take some time to inspect the bar charts shown below for positions A, B, D, F and G. What do you notice?

When you inspect the bar charts, it is evident that as the bob moves from A to D, the kinetic energy is increasing and the potential energy is decreasing. However, the total amount of these two forms of energy is remaining constant. Whatever potential energy is lost in going from position A to position D appears as kinetic energy. There is a transformation of potential energy into kinetic energy as the bob moves from position A to position D. Yet the total mechanical energy remains constant. We would say that mechanical energy is conserved. As the bob moves past position D towards position G, the opposite is observed. Kinetic energy decreases as the bob moves rightward and (more importantly) upward toward position G. There is an increase in potential energy to accompany this decrease in kinetic energy. Energy is being transformed from kinetic form into potential form. Yet, as illustrated by the **TME** bar, the total amount of mechanical energy is conserved. This very principle of energy conservation was explained in the Energy chapter of The Physics Classroom Tutorial.

# The Period of a Pendulum

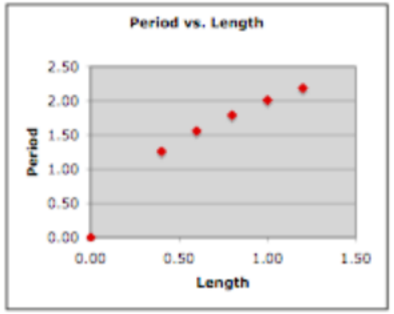
Our final discussion will pertain to the period of the pendulum. As discussed previously in this lesson, the period is the time it takes for a vibrating object to complete its cycle. In the case of pendulum, it is the time for the pendulum to start at one *extreme*, travel to the opposite *extreme*, and then return to the original location. Here we will be interested in the question *What variables affect the period of a pendulum?* We will concern ourselves with possible variables. The variables are the mass of the pendulum bob, the length of the string on which it hangs, and the *angular displacement*. The angular displacement or *arc angle* is the angle that the string makes with the vertical when released from rest. These three variables and their effect on the period are easily studied and are often the focus of a physics lab in an introductory physics class. The data table below provides representative data for such a study.

| Trial | Mass (kg) | Length (m) | Arc Angle (°) | Period (s) |
|---|---|---|---|---|
| 1 | 0.02- | 0.40 | 15.0 | 1.25 |
| 2 | 0.050 | 0.40 | 15.0 | 1.29 |
| 3 | 0.100 | 0.40 | 15.0 | 1.28 |
| 4 | 0.200 | 0.40 | 15.0 | 1.24 |
| 5 | 0.500 | 0.40 | 15.0 | 1.26 |
| 6 | 0.200 | 0.60 | 15.0 | 1.56 |
| 7 | 0.200 | 0.80 | 15.0 | 1.79 |
| 8 | 0.200 | 1.00 | 15.0 | 2.01 |
| 9 | 0.200 | 1.20 | 15.0 | 2.19 |
| 10 | 0.200 | 0.40 | 10.0 | 1.27 |
| 11 | 0.200 | 0.40 | 20.0 | 1.29 |
| 12 | 0.200 | 0.40 | 25.0 | 1.25 |
| 13 | 0.200 | 0.40 | 30.0 | 1.26 |

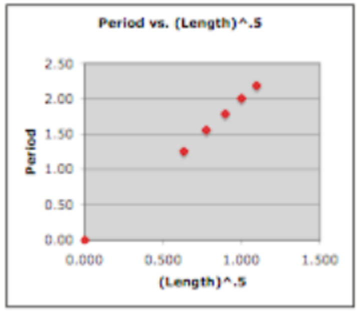
In trials 1 through 5, the mass of the bob was systematically altered while keeping the other quantities constant. By so doing, the experimenters were able to investigate the possible effect of the mass upon the period. As can be seen in these five trials, alterations in mass have little effect upon the period of the pendulum.In trials 4 and 6-9, the mass is held constant at 0.200 kg and the arc angle is held constant at 15°. However, the length of the pendulum is varied. By so doing, the experimenters were able to investigate the possible effect of the length of the string upon the period. As can be seen in these five trials, alterations in length definitely have an effect upon the period of the pendulum. As the string is lengthened, the period of the pendulum is increased. There is a direct relationship between the period and the length.

Finally, the experimenters investigated the possible effect of the arc angle upon the period in trials 4 and 10-13. The mass is held constant at 0.200 kg and the string length is held constant at 0.400 m. As can be seen from these five trials, alterations in the arc angle have little to no effect upon the period of the pendulum.

So the conclusion from such an experiment is that the one variable that effects the period of the pendulum is the length of the string. Increases in the length lead to increases in the period. But the investigation doesn't have to stop there. The quantitative equation relating these variables can be determined if the data is plotted and linear regression analysis is performed. The two plots below represent such an analysis. In each plot, values of period (the dependent variable) are placed on the vertical axis. In the plot on the left, the length of the pendulum is placed on the horizontal axis. The shape of the curve indicates some sort of power relationship between period and length. In the plot on the right, the square root of the length of the pendulum (length to the ½ power) is plotted. The results of the regression analysis are shown.




Slope: 1.7536
Y-intercept: 0.2616
COR: 0.9183




Slope: 2.0045
Y-intercept: 0.0077
COR: 0.9999

The analysis shows that there is a better fit of the data and the regression line for the graph on the right. As such, the plot on the right is the basis for the equation relating the period and the length. For this data, the equation is

$$\text{Period} = 2.0045 \bullet \text{Length}^{0.5} + 0.0077$$

Using **T** as the symbol for period and **L** as the symbol for length, the equation can be rewritten as

$$T = 2.0045 \bullet L^{0.5} + 0.0077$$

The commonly reported equation based on theoretical development is

$$T = 2 \bullet \Pi \bullet (L/g)^{0.5}$$

where **g** is a constant known as the gravitational field strength or the acceleration of gravity (9.8 N/kg). The value of 2.0045 from the experimental investigation agrees well with what would be expected from this theoretically reported equation. Substituting the value of **g** into this equation, yields a proportionality constant of $2\Pi/g^{0.5}$, which is 2.0071, very similar to the 2.0045 proportionality constant developed in the experiment.

# Motion of a Mass on a Spring

In a previous part of this lesson, the motion of a mass attached to a spring was described as an example of a vibrating system. The mass on a spring motion was discussed in more detail as we sought to understand the mathematical properties of objects that are in periodic motion. Now we will investigate the motion of a mass on a spring in even greater detail as we focus on how a variety of quantities change over the course of time. Such quantities will include forces, position, velocity and energy - both kinetic and potential energy.

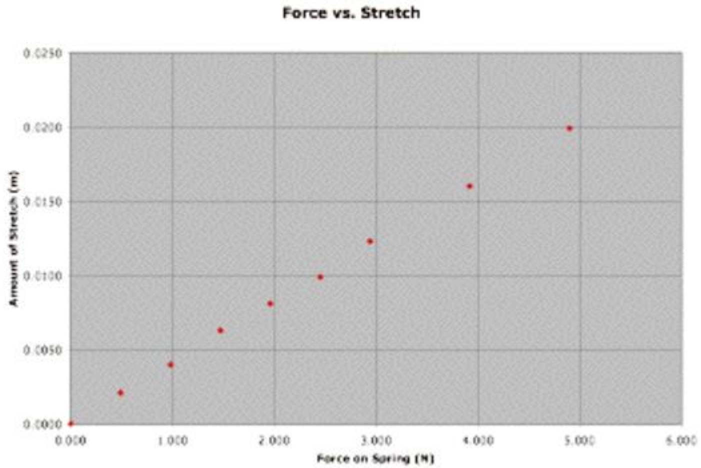
# Hooke's Law

We will begin our discussion with an investigation of the forces exerted by a spring on a hanging mass. Consider the system shown at the right with a spring attached to a support. The spring hangs in a relaxed, unstretched position. If you were to hold the bottom of the spring and pull downward, the spring would stretch. If you were to pull with just a little force, the spring would stretch just a little bit. And if you were to pull with a much greater force, the spring would stretch a much greater extent. Exactly what is the quantitative relationship between the amount of pulling force and the amount of stretch?

To determine this quantitative relationship between the amount of force and the amount of stretch, objects of known mass could be attached to the spring. For each object which is added, the amount of stretch could be measured. The force which is applied in each instance would be the weight of the object. A regression analysis of the force-stretch data could be performed in order to determine the quantitative relationship between the force and the below shows some repres

| Mass (kg) | Force on Spring (N) | Amount of Stretch (m) |
|---|---|---|
| 0.000 | 0.000 | 0.0000 |
| 0.050 | 0.490 | 0.0021 |
| 0.100 | 0.980 | 0.0040 |
| 0.150 | 1.470 | 0.0063 |
| 0.200 | 1.960 | 0.0081 |
| 0.250 | 2.450 | 0.0099 |
| 0.300 | 2.940 | 0.0123 |
| 0.400 | 3.920 | 0.0160 |
| 0.500 | 4.900 | 0.0199 |

By plotting the force-stretch data and performing a linear regression analysis, the quantitative relationship or equation can be determined. The plot is shown below.



A linear regression analysis yields the following statistics:

slope = 0.00406 m/N
y-intercept = 3.43 x$10^{-5}$ (*pert* near close to 0.000)
regression constant = 0.999

The equation for this line is

$$\text{Stretch} = 0.00406 \bullet \text{Force} + 3.43\text{x}10^{-5}$$

BİRUNİ ÜNİVERSİTESİ "Bilimin Geleceği"

The fact that the regression constant is very close to 1.000 indicates that there is a *strong fit* between the equation and the data points. This *strong fit* lends credibility to the results of the experiment.

This relationship between the force applied to a spring and the amount of stretch was first discovered in 1678 by English scientist Robert Hooke. As Hooke put it: *Ut tensio, sic vis*. Translated from Latin, this means "As the extension, so the force." In other words, the amount that the spring extends is proportional to the amount of force with which it pulls. If we had completed this study about 350 years ago (and if we knew some Latin), we would be famous! Today this quantitative relationship between force and stretch is referred to as Hooke's law and is often reported in textbooks as

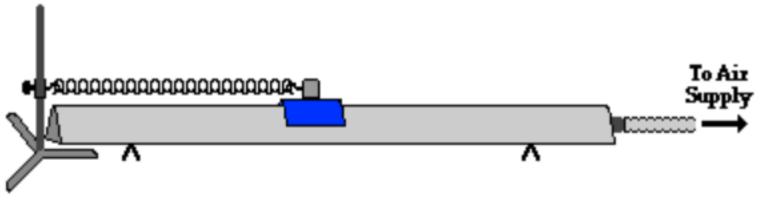
$$F_{spring} = -k \bullet x$$

where **Fspring** is the force exerted upon the spring, **x** is the amount that the spring stretches relative to its relaxed position, and **k** is the proportionality constant, often referred to as the spring constant. The spring constant is a positive constant whose value is dependent upon the spring which is being studied. A stiff spring would have a high spring constant. This is to say that it would take a relatively large amount of force to cause a little displacement. The units on the spring constant are Newton/meter (N/m). The negative sign in the above equation is an indication that the direction that the spring stretches is opposite the direction of the force which the spring exerts. For instance, when the spring was stretched below its relaxed position, x is *downward*. The spring responds to this stretching by exerting an *upward* force. The x and the F are in opposite directions. A final comment regarding this equation is that it works for a spring which is stretched vertically and for a spring is stretched horizontally (such as the one to be discussed below).

# Force Analysis of a Mass on a Spring

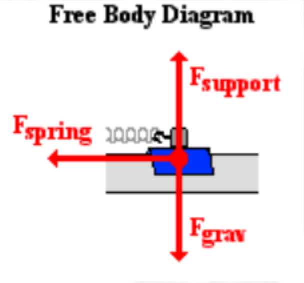
Earlier in this lesson we learned that an object that is vibrating is acted upon by a restoring force. The restoring force causes the vibrating object to slow down as it moves away from the equilibrium position and to speed up as it approaches the equilibrium position. It is this restoring force which is responsible for the vibration. So what is the restoring force for a mass on a spring?

We will begin our discussion of this question by considering the system in the diagram below.

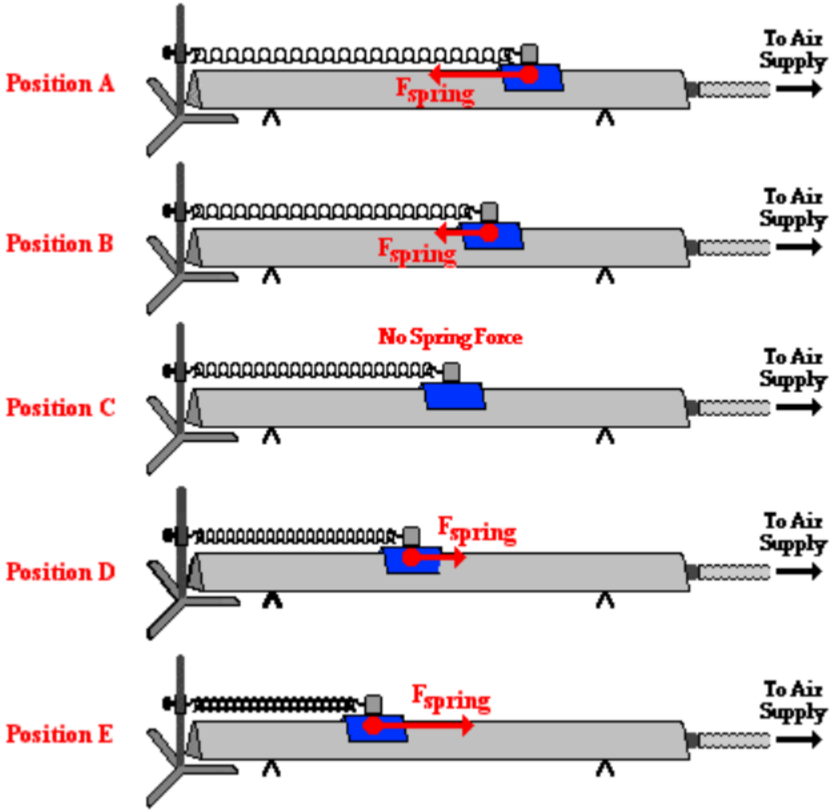


of its magnitude and its direction. The force of gravity always acts downward; its magnitude can be

The diagram shows an air track and a glider. The glider is attached by a spring to a vertical support. There is a negligible amount of friction between the glider and the air track. As such, there are three dominant forces acting upon the glider. These three forces are shown in the free-body diagram at the right. The force of gravity (**Fgrav**) is a rather predictable force - both in terms



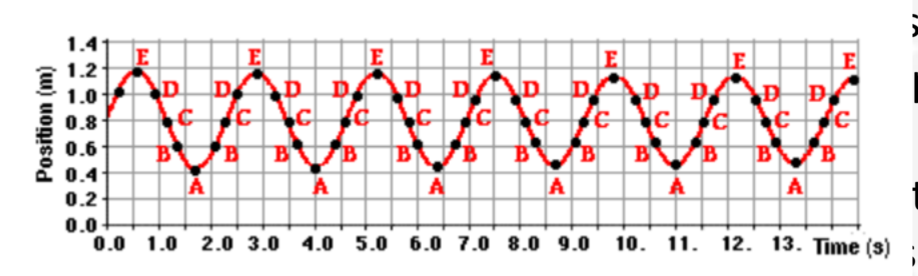

found as the product of mass and the acceleration of gravity (**m•9.8 N/kg**). The support force (**Fsupport**) balances the force of gravity. It is supplied by the air from the air track, causing the glider to *levitate* about the track's surface. The final force is the spring force (**Fspring**). As discussed above, the spring force varies in magnitude and in direction. Its magnitude can be found using Hooke's law. Its direction is always opposite the direction of stretch and towards the equilibrium position. As the air track glider does *the back and forth*, the spring force (**Fspring**) acts as the restoring force. It acts leftward on the glider when it is positioned to the right of the equilibrium positions over the course of the glider's path. As the glider moves from position A (the release point) to position B and then to position C, the spring force acts leftward upon the leftward moving glider. As the glider approaches position C, the amount of stretch of the spring decreases and the spring force decreases, consistent with Hooke's Law. Despite this decrease in the spring force, there is still an acceleration caused by the restoring force for the entire span from position A to position C. At position C, the glider has reached its maximum speed. Once the glider passes to the left of position C, the spring force acts rightward. During this phase of the glider's cycle, the spring is being compressed. The further past position C that the glider moves, the greater the amount of compression and the greater the spring force. This spring force acts as a restoring force, slowing the glider down as it moves from position C to position D to position E. By the time the glider has reached position E, it has slowed down to a momentary rest position before changing its direction and heading back towards the equilibrium position. During the glider's motion from position E to position C, the amount that the spring is compressed decreases and the spring force decreases. There is still an acceleration for the entire distance from position E to position C. At position C, the glider has reached its maximum speed. Now the glider begins to move to the right of point C. As it does, the spring force acts leftward upon the rightward moving



Position A
To Air Supply
Fspring
Position B
To Air Supply
Fspring
No Spring Force
Position C
To Air Supply
Position D
Fspring
To Air Supply
Position E
Fspring
To Air Supply

# Sinusoidal Nature of the Motion of a Mass on a Spring

Previously in this lesson, the variations in the position of a mass on a spring with respect to time were discussed. At that time, it was shown that the position of a mass on a spring varies with the sine




; that was vibr... pring. The disc... moving along th... the right enc... . time plot, the plot would look like the plot below. Position A is the right-most position on the air track when the glider is closest to the detector.

The labeled positions in the diagram above are the same

position . You

might re were

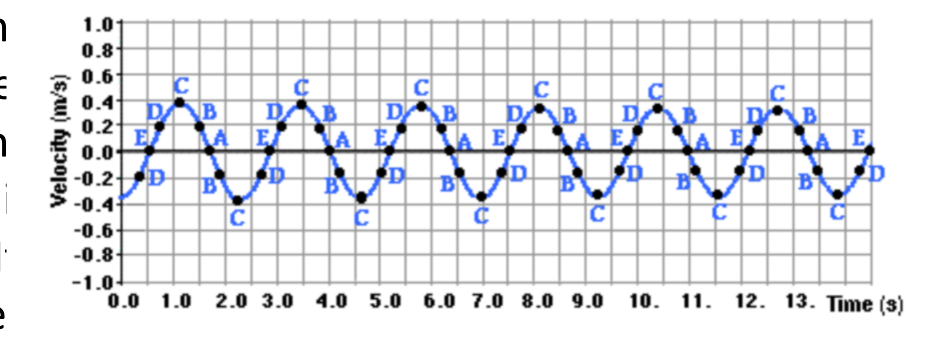
position C was

the equi um

speed. I on-time

data we otted

data would look like the graph below.



Observe that the velocity-time plot for the mass on a spring is also a sinusoidal shaped plot. The only difference between the position-time and the velocity-time plots is that one is shifted one-fourth of a vibrational cycle away from the other. Also observe in the plots that the absolute value of the velocity is greatest at position C (corresponding to the equilibrium position). The velocity of any moving object, whether vibrating or not, is the speed with a direction. The magnitude of the velocity is the speed. The direction is often expressed as a positive or a negative sign. In some instances, the velocity has a negative direction (the glider is moving leftward) and its velocity is plotted below the time axis. In other cases, the velocity has a positive direction (the glider is moving rightward) and its velocity is plotted above the time axis. You will also notice that the velocity is zero whenever the position is at an extreme. This occurs at positions A and E when the glider is beginning to change direction. So just as in the case of pendulum motion, the speed is greatest when the displacement of the mass relative to its equilibrium position is the least. And the speed is least when the displacement of the mass relative to its equilibrium position is the greatest.

# Energy Analysis of a Mass on a Spring

On the previous page, an energy analysis for the vibration of a pendulum was discussed. Here we will conduct a similar analysis for the motion of a mass on a spring. In our discussion, we will refer to the motion of the frictionless glider on the air track that was introduced above. The glider will be pulled to the right of its equilibrium position and be released from rest (position A). As mentioned, the glider then accelerates towards position C (the equilibrium position). Once the glider passes the equilibrium position, it begins to slow down as the spring force pulls it backwards against its motion. By the time it has reached position E, the glider has slowed down to a momentary pause before changing directions and accelerating back towards position C. Once again, after the glider passes position C, it begins to slow down as it approaches position A. Once at position A, the cycle begins all over again ... and again ... and again.

The kinetic energy possessed by an object is the energy it possesses due to its motion. It is a quantity that depends upon both mass and speed. The equation that relates kinetic energy (**KE**) to mass (**m**) and speed (**v**) is

$$KE = \frac{1}{2} \bullet m \bullet v^2$$

The faster an object moves, the more kinetic energy that it will possess. We can combine this concept with the discussion above about how speed changes during the course of motion. This blending of the concepts would lead us to conclude that the kinetic energy of the mass on the spring increases as it approaches the equilibrium position; and it decreases as it moves away from the equilibrium position.

This information is summarized in the table below:

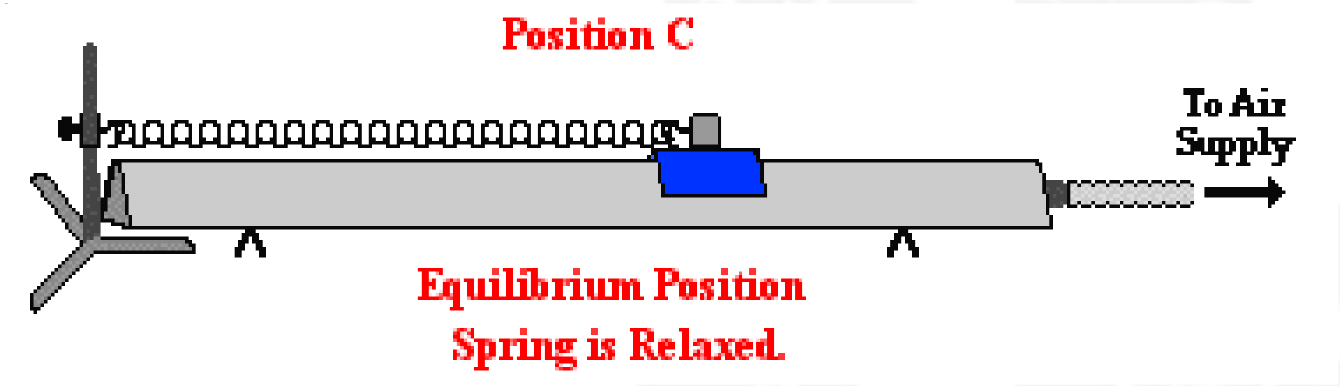
| Stage of Cycle | Change in Speed | Change in Kinetic Energy |
|---|---|---|
| A to B to C | Increasing | Increasing |
| C to D to E | Decreasing | Decreasing |
| E to D to C | Increasing | Increasing |
| C to B to A | Decreasing | Decreasing |

Kinetic energy is only one form of mechanical energy. The other form is potential energy. Potential energy is the stored energy of position possessed by an object. The potential energy could be gravitational potential energy, in which case the position refers to the height above the ground. Or the potential energy could be **elastic potential energy**, in which case the position refers to the position of the mass on the spring relative to the equilibrium position. For our vibrating air track glider, there is no change in height. So the gravitational potential energy does not change. This form of potential energy is not of much interest in our analysis of the energy changes. There is however a change in the position of the mass relative to its equilibrium position. Every time the spring is compressed or stretched relative to its relaxed position, there is an increase in the elastic potential energy. The amount of elastic potential energy depends on the amount of stretch or compression of the spring. The equation that relates the amount of elastic potential energy (**PEspring**) to the amount of compression or stretch (**x**) is

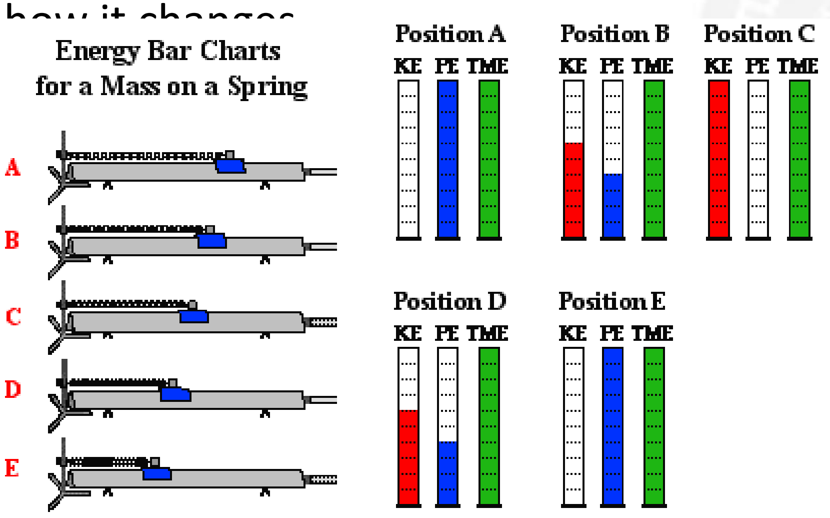
$$PE_{spring} = \frac{1}{2} \bullet k \bullet x^2$$

where **k** is the spring constant (in N/m) and **x** is the dist that the spring is stretched or compressed relative to th relaxed, unstretched position.

When the air track glider is at its equilibrium position (position C), it is moving it's fastest (as discussed above). At this position, the value of x is 0 meter. So the amount of elastic potential energy (**PEspring**) is 0 Joules. This is the position where the potential energy is the least. When the gl
the spring is stretched the greatest distan
potential energy is a maximum. A similar
made for position E. At position E, the spr
most and the elastic potential energy at t
maximum. Since the spring stretches as m
the elastic potential energy at position A
(the *stretched* position) is the same as at position E (the *compressed* position). At these two positions - A and E - the velocity is 0 m/s and the kinetic energy is 0 J. So just like the case of a vibrating pendulum, a vibrating mass on a spring has the greatest potential energy when it has the smallest kinetic energy. And it also has the smallest potential energy (position C) when it has the greatest kinetic energy. These principles are shown in the animation below.



When conducting an energy analysis, a common representation is an energy bar chart. An energy bar chart uses a bar graph to represent the relative amount and form of energy possessed by an object as it is moving. It is a useful conceptual tool for showing what form of energy is present and
over the course of time. The diagram below
chart for the air track glider and spring syste



The bar chart reveals that as the mass on the spring moves from A to B to C, the kinetic energy increases and the elastic potential energy decreases. Yet the total amount of these two forms of mechanical energy remains constant. Mechanical energy is being transformed from potential form to kinetic form; yet the total amount is being *conserved*. A similar conservation of energy phenomenon occurs as the mass moves from C to D to E. As the spring becomes compressed and the mass slows down, its kinetic energy is transformed into elastic potential energy. As this transformation occurs, the total amount of mechanical energy is conserved. This very principle of energy conservation was explained in a previous chapter - the Energy chapter - of The Physics Classroom Tutorial.

# Period of a Mass on a Spring

As is likely obvious, not all springs are created equal. And not all spring-mass systems are created equal. One measurable quantity that can be used to distinguish one spring-mass system from another is the period. As discussed earlier in this lesson, the period is the time for a vibrating object to make one complete cycle of vibration. The variables that effect the period of a spring-mass system are the mass and the spring constant. The equation that relates these variables resembles the equation for the period of a pendulum. The equation is

$$T = 2 \bullet \Pi \bullet (m/k)^{.5}$$

where **T** is the period, **m** is the mass of the object attached to the spring, and **k** is the spring constant of the spring. The equation can be interpreted to mean that more massive objects will vibrate with a longer period. Their greater inertia means that it takes more time to complete a cycle. And springs with a greater spring constant (stiffer springs) have a smaller period; masses attached to these springs take less time to complete a cycle. Their greater spring constant means they exert stronger restoring forces upon the attached mass. This greater force reduces the length of time to complete one cycle of vibration.

# Looking Forward to Lesson 2